2017-11-1

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Week #6 & #7

Time Response……………………………………………EE362

Definitions

- The **zeros** of transfer function are the values of the laplace transform variable, s, that cause the transfer function to become **zero**.
- The **poles** of transfer function are the values of the laplace transform variable, s, that cause the transfer function to become **infinity**.

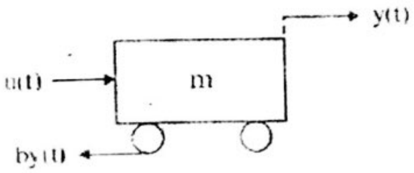
$$G(s) = \frac{s+5}{s^2+5s+6}$$ zero : -5 , poles: -2 , -3.

- The output response of a system is the sum of two responses: **Forces Response and the Natural Response.** The forced response is also called the steady-state response, or particular solution. The natural response is also called the homogeneous solution .
- The order of a system is defined as being the highest power of derivative in the differential equation, or being the highest power of s in the denominator of the transfer function.

First order system $G(s) = \frac{5}{s+3}$, Second order system $G(s) = \frac{5}{s^2+6s+8}$

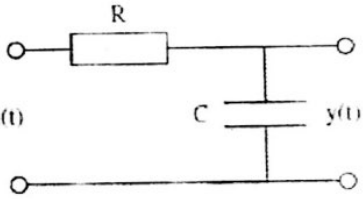
First order system

Mechanical system



$$m\frac{dy(t)}{dt} + by(t) = u(t)$$

Electrical system



$$RC\frac{dy(t)}{dt} + y(t) = u(t)$$

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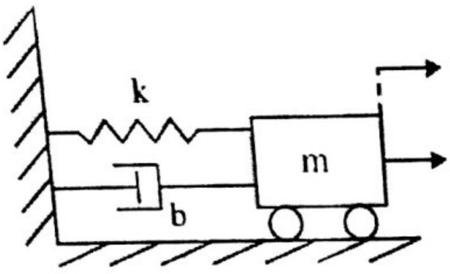
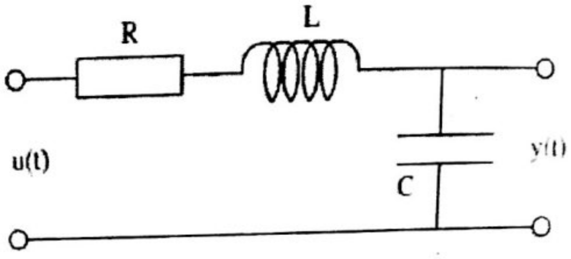
Second order system

Mechanical system



$$m\frac{d^2y(t)}{dt^2}+b\frac{dy(t)}{dt}+ky(t)=u(t)$$

Electrical system



$$LC\frac{d^2y(t)}{dt^2}+RC\frac{dy(t)}{dt}+y(t)=u(t)$$

- The time response of a control system consists of two parts: **the transient response** and the **steady state response**. The transient response is defined as the part of the time response which goes from the initial state to the final state. The steady-state response is defined as the behavior of the system as **t** approaches **infinity** after the transients have died out. Thus the system response y(t) may be written as:
-

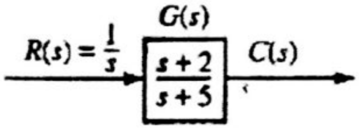
$$y(t) = y_t(t) + y_{ss}(t)$$

Where $y_t(t)$ denoted transient response and $y_{ss}(t)$ denoted steady state response.

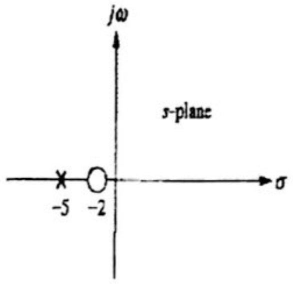
- A first-order system only has s to the power one in the denominator, while a second-order system has the highest power of s in the denominator being two.

- Types of the input functions (or test input signals) commonly are:
  Impulse function - step function - Ramp function - Sinusoidal function.

Ex: Given the system in figure below, write the output, c(t), in general terms. Specify the forced and natural parts of the solution.



Pole zero map.



System response (step response)

$$C(s) = \frac{(s+2)}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} = \frac{2/5}{s} + \frac{3/5}{s+5}$$

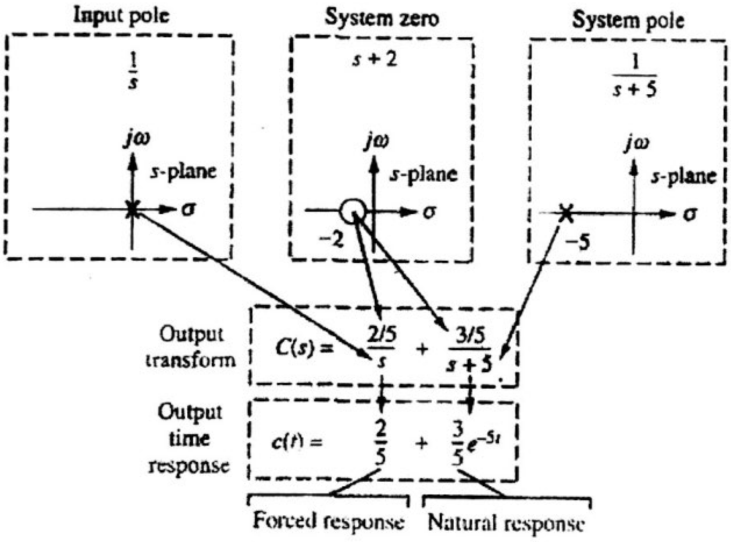
where

$$A = \left.\frac{(s+2)}{(s+5)}\right|_{s\to 0} = \frac{2}{5}$$

$$B = \left.\frac{(s+2)}{s}\right|_{s\to -5} = \frac{3}{5}$$

Thus,

$$c(t) = \frac{2}{5} + \frac{3}{5}e^{-5t}$$



From the development summarized in last figure, we draw the following conclusions:

1. A pole of the input function generates the form of the forced response (that is, the pole at the origin generated a step function at the output).

2. A pole of the transfer function generates the form of the natural response (that is, the pole at -5 generated e^{-5t}).

3. A pole on the real axis generates an exponential response of the form e^{-at}, where -a is the pole location on the real axis. Thus, the farther to the left a pole is on the negative real axis, the faster the exponential transient response will decay to zero.

4. The zeros and poles generate the amplitudes for both the forced and natural responses .

Exercise: What are the orders of the systems described by the following transfer functions:

a) $G(s)=\dfrac{1}{ms^2+bs+k}$

b) $G(s)=\dfrac{1}{RCs+1}$

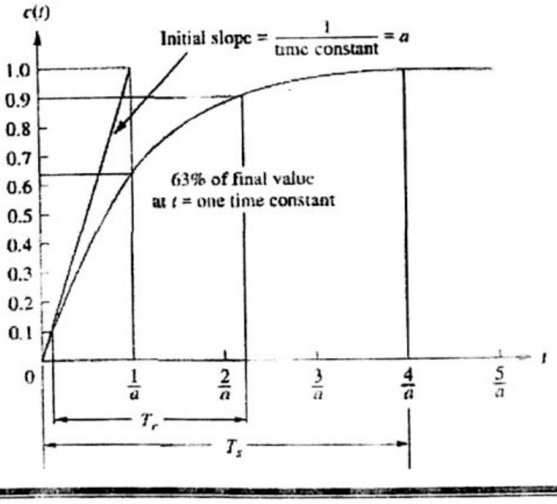
c) $G(s)=\dfrac{1}{LCs^2+RCs+1}$

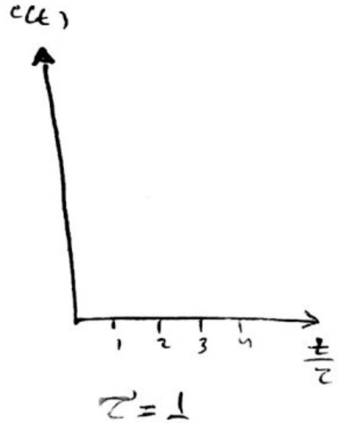
First order system

The general form of a transfer function for a first-order system is

$$G(s)=\frac{a}{s+a}=\frac{1}{\tau s+1}$$

If the input is a unit step, where R(s) = 1/s.





$\tau = \frac{1}{a}$

$$C(s) = R(s)G(s) = \frac{a}{s(s+a)}$$

Taking the inverse transform, the step response is given by

$$\boxed{c(t) = c_f(t) + c_n(t) = 1 - e^{-at}}$$

When $t = 1/a$,

$$e^{-at}|_{t=1/a} = e^{-1} = 0.37$$

$$c(t)|_{t=1/a} = 1 - e^{-at}|_{t=1/a} = 1 - 0.37 = 0.63$$

Transient Response Specification

Time constant (T_c) associated with this system is $\tau = 1/a$. The time constant tells how quickly the system responds. So, systems that respond quickly have *large* values of a, and systems that respond slowly have *small* values of a.

Rise time Tr, is defined as the time for the waveform to go from 10% to 90% of its final value.

$$T_r = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a}$$

Settling timeTs is defined as the time for the response to reach, and stay within, 2% of its final value.

$$T_s = \frac{4}{a} = 4\tau$$

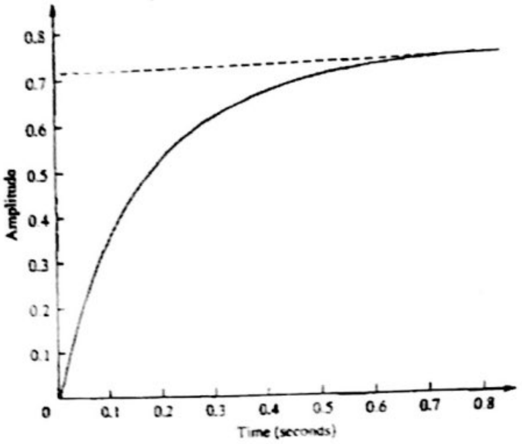
Steady state error(ess) The steady-state error is the error after the transient response has decayed leaving only the continuous response. The error signal:

$$e(t) = r(t) - y(t) = 1 - (1 - e^{-at}) = e^{-at}$$

As t approaches infinity, e^{-at} approaches zero and the steady state error is :

$$e_{ss} = e(\infty) = \lim_{t \to 0}[r(t) - y(t)] = 0$$

Ex Consider a simple first order system G(s) = K/(s+a). Assume the unit step response given in figure below. Determine the value of a , K and first order characteristic.



The step response

$$C(s) = \frac{K}{s(s+a)} = \frac{K/a}{s} - \frac{K/a}{(s+a)}$$

$$c(t) = \frac{k}{a} - \frac{k}{a}\, e^{-at}$$

From the response, we measure the time constant, that is, the time for the amplitude to reach 63% of its final value. Since the final value is about 0.72. The time constant is evaluated whe the curve reaches 0.63 x 0.72 = 0.45.

From the curve when

$$c(t) = 0.45 \quad \ldots\ldots\ldots\ldots\ t = Tc = 0.13 \text{ second.}$$

$$Tc = \frac{1}{a} = 0.13 \qquad \rightarrow \qquad a = \frac{1}{T_c} = \frac{1}{0.13} = 7.7$$

Rise time

$$Tr = \frac{2.2}{a} = \frac{2.2}{7.7} = 0.286\ s$$

Settling time

$$Ts = 4\,Tc = \frac{4}{a} = \frac{4}{7.7} = 0.52\ s$$

To fine K, the final value (steady state value)

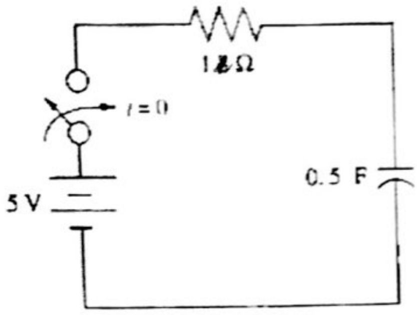
$$c(\infty) = 0.72 = \frac{k}{a} - \frac{k}{a}\, e^{-a\infty} = \frac{K}{a} = \frac{K}{7.7} \qquad \rightarrow K = 7.7 * 0.72 = 5.54$$

The transfer function

$$G(s) = \frac{5.54}{s+7.7}$$

Ex

Find the capacitor voltage in the network shown in figure below if the switch closes at t = 0. Assume zero initial conditions. Also find the time constant, rise time, and settling time for the capacitor voltage.



Using voltage division, $\frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{(R+\frac{1}{Cs})} = \frac{2}{(s+2)}$. Since $V_i(s) = \frac{5}{s}$, $V_C(s) = \frac{10}{s(s-2)} = \frac{5}{s} - \frac{5}{s+2}$

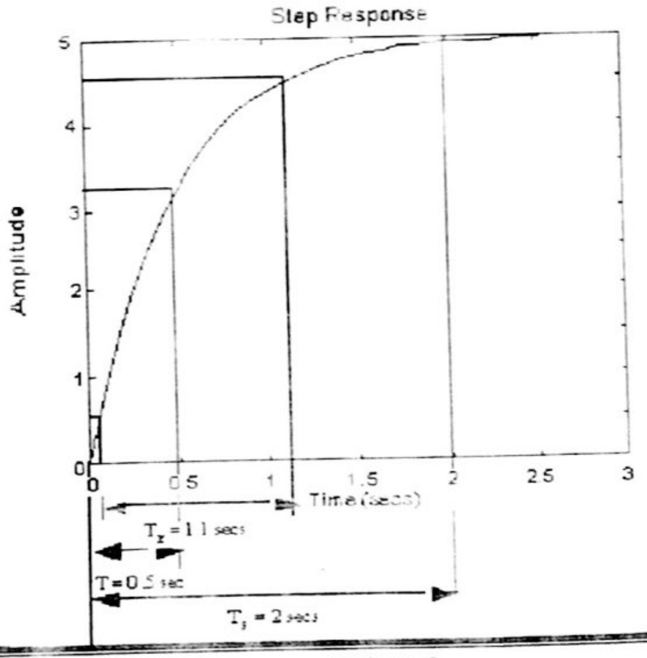
Therefore $v_C(t) = 5 - 5e^{-2t}$. Also, $T = \frac{1}{2}$, $T_r = \frac{2.2}{a} = \frac{2.2}{2} = 1.1$, $T_s = \frac{4}{a} = \frac{4}{2} = 2$.

Program:

```
clf   .
num=2;
den=[1 2];
G=tf(num,den)
step(5*G)
```

Computer response:

```
Transfer function:
   2
-----
s + 2
```

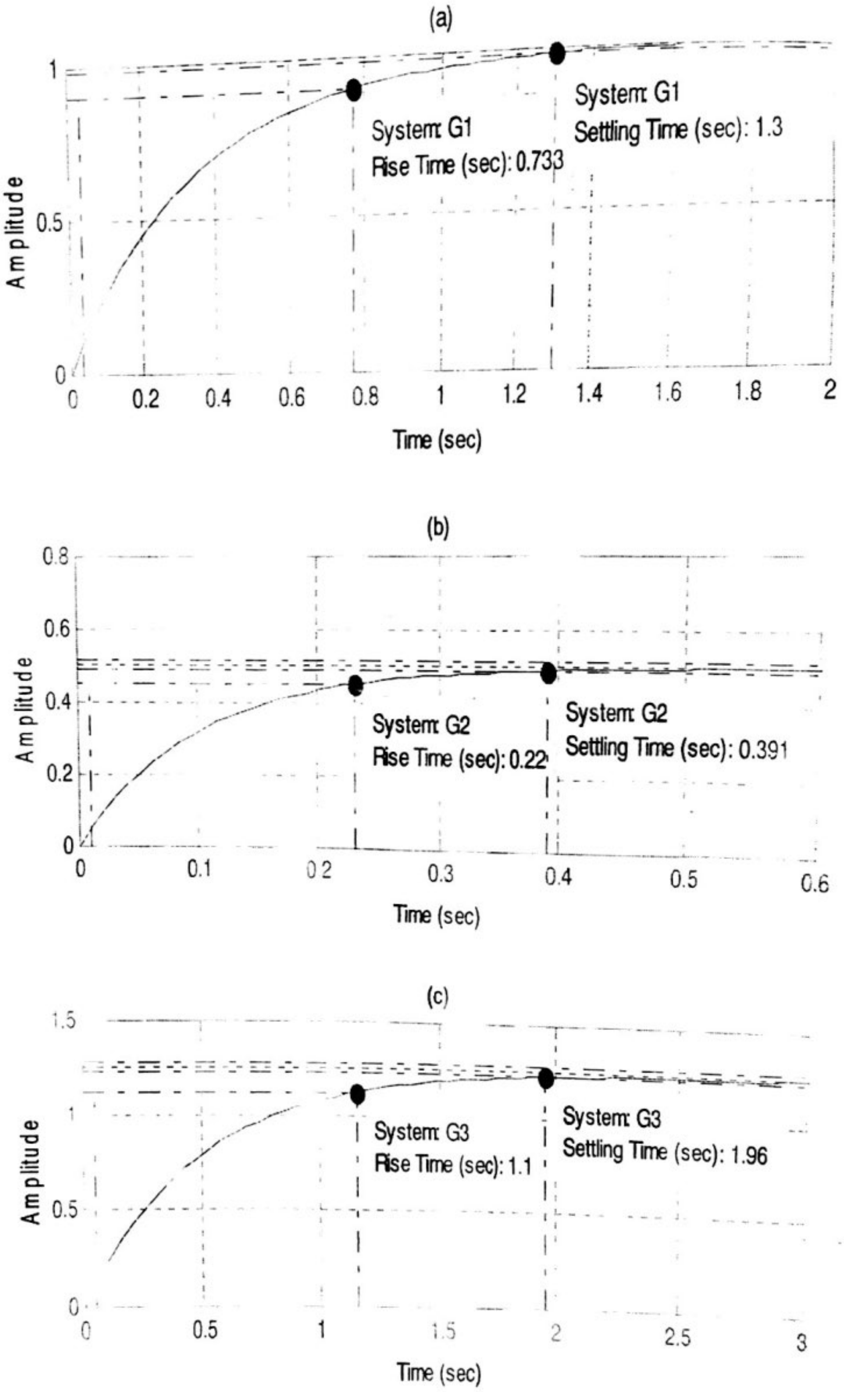


Ex :
Plot the time responses and find Tr ,Ts for the following systems using MATLAB .

$$G1(s) = \frac{3}{s+3} \quad , \quad G2(s) = \frac{5}{s+10} \quad , G3(s) = \frac{5}{2s+4}$$

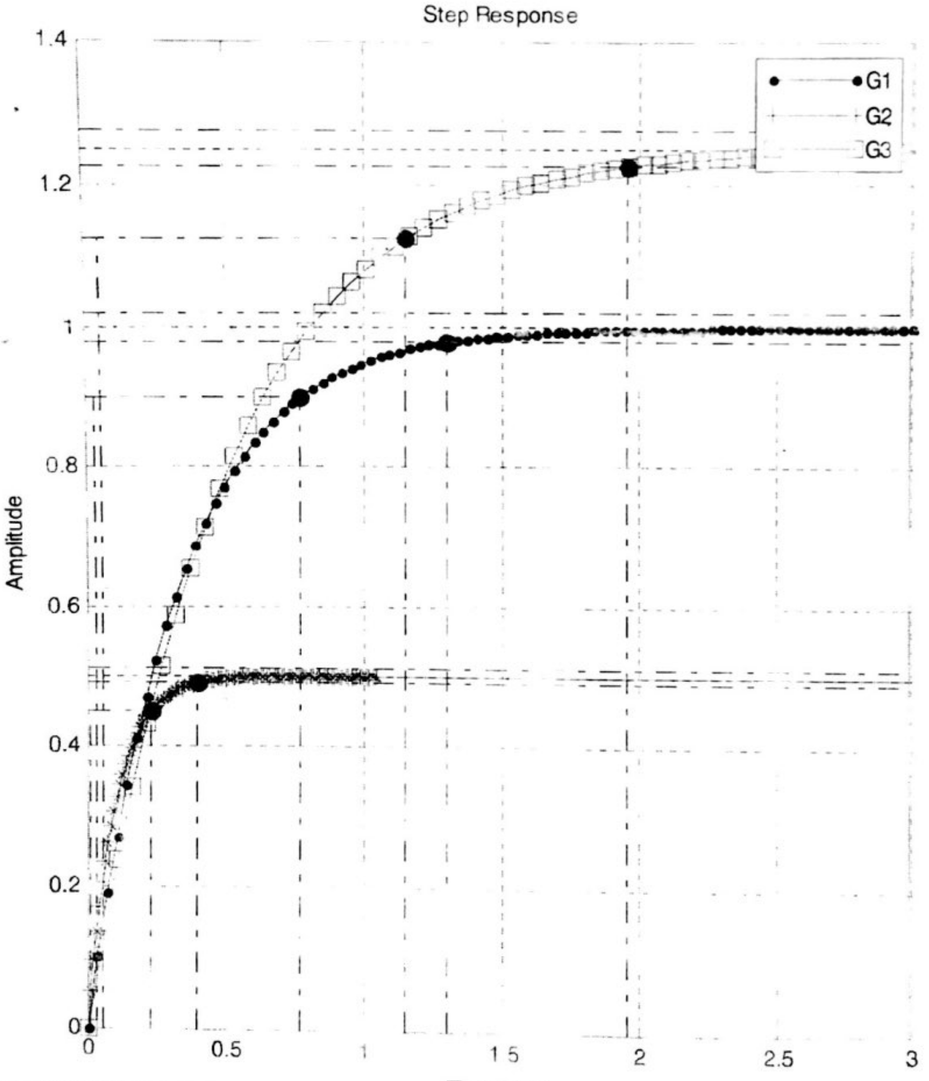
MATLAB code1

```
num1=3;
den1=[1 3];
G1=tf(num1,den1)
subplot(3,1,1)
step(G1)
title('(a)')
grid
num2=5;
den2=[1 10];
G2=tf(num2,den2)
subplot(3,1,2)
step(G2)
title('(b)')
grid
num3=5;
den3=[2 4];
G3=tf(num3,den3)
subplot(3,1,3)
step(G3)
title('(c)')
grid
```



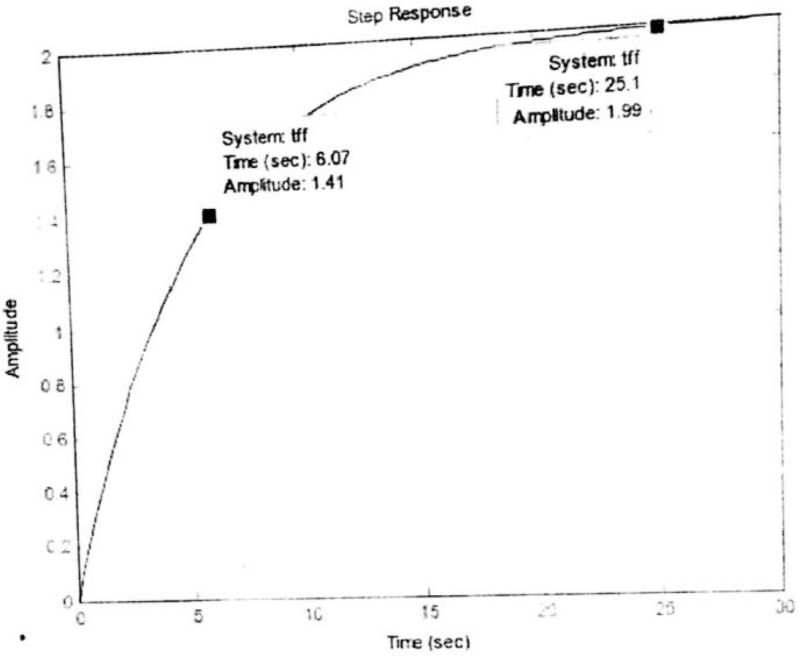
MATLAB code2

```
num1=3;
den1=[1 3];
G1= tf(num1,den1)
num2=5;
den2=[1 10];
G2= tf(num2,den2)
num3=5;
den3=[2 4];
G3= tf(num3,den3)
step(G1,G2,G3)
legend('G1  ', 'G2','G3')
grid
```

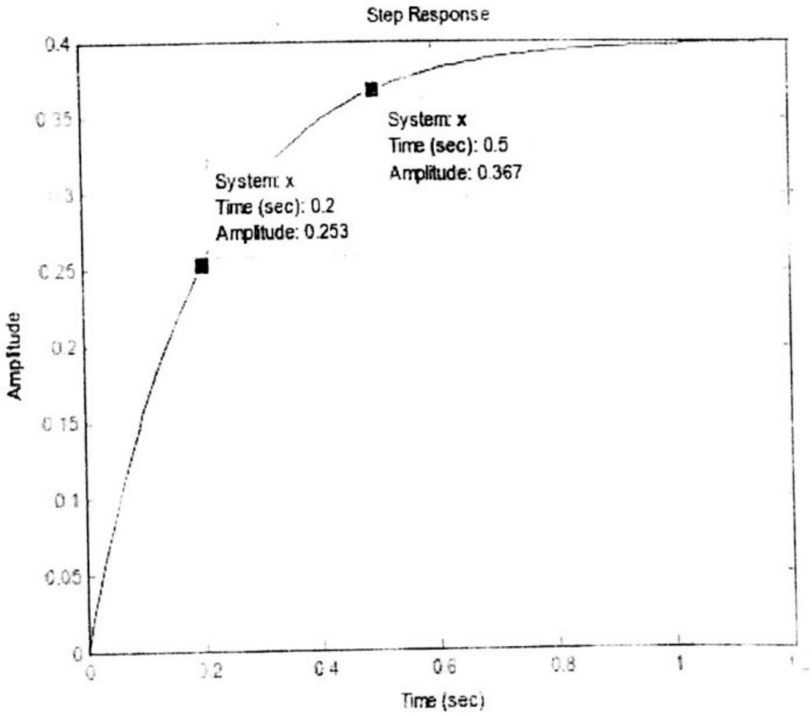


Ex:

The following figures present the step responses (to a unit step) of first order systems. Determine the transfer functions of the two systems. To help with the calculations, some points were given explicitly on the graph.



Step response of a system



Step response of a system

Second order system

A general form of a second order system is:

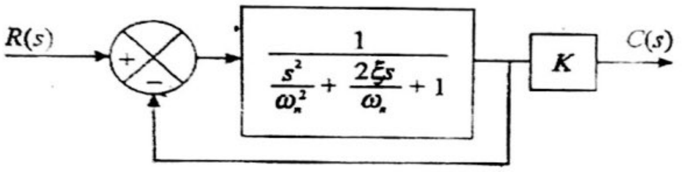
$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n \frac{dy(t)}{dt} + \omega_n^2 y(t) = k\omega_n^2 u(t)$$

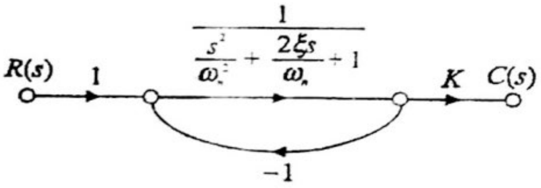
Transfer function:

$$\boxed{\frac{Y(s)}{R(s)} = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

k : the gain of the system
ζ : the damping ratio of the system
$\boldsymbol{\omega_n}$: the (undamped) natural frequency of the system.

The block diagram and signal flow graph representations of a second order system are given in figure below





assume k=1, for simplicity

$$\boxed{\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

The **characteristic equation** for the system is

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

Therefore, its roots are given by

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$S_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} = -\sigma_d \pm j\omega_d$$

$\boldsymbol{\sigma_d}$ **:** Magnitude of the real part of the pole and is the ***exponential damping frequency***.
$\boldsymbol{\omega_d}$ **:** Imaginary part of the pole and is called ***damped frequency of oscillation***.

Step response for the general second-order system

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where it is assumed that $\zeta < 1$ (the underdamped case). Expanding by partial fractions,

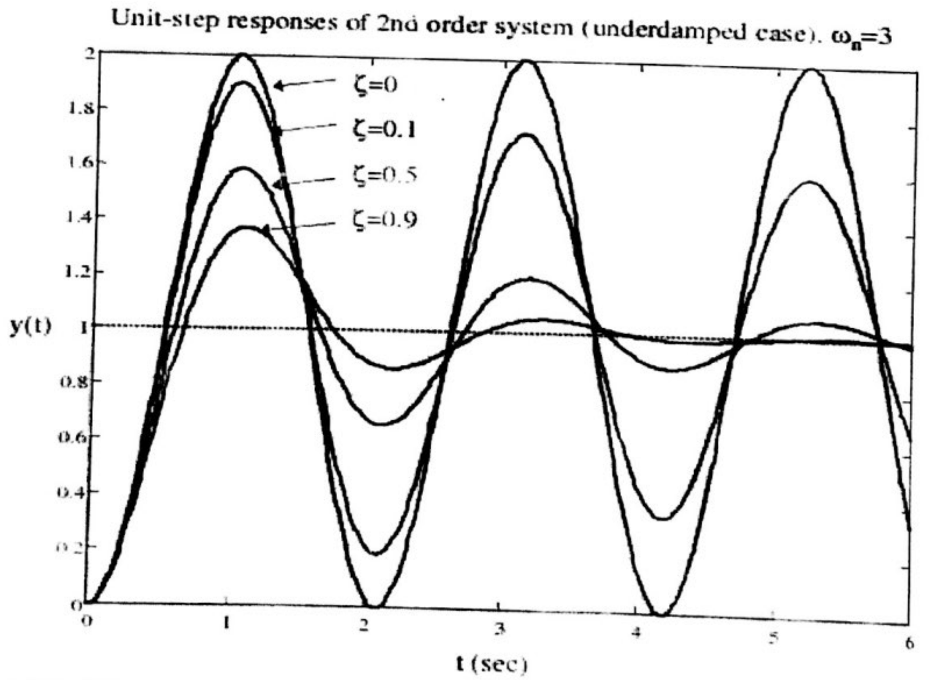
$$C(s) = \frac{1}{s} - \frac{(s + \zeta\omega_n) + \frac{\zeta}{\sqrt{1-\zeta^2}}\omega_n\sqrt{1-\zeta^2}}{(s + \zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

Taking the inverse Laplace transform, which is left as an exercise for the student, produces

$$\boxed{\begin{aligned} c(t) &= 1 - e^{-\zeta\omega_n t}\left(\cos \omega_n\sqrt{1-\zeta^2}t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin \omega_n\sqrt{1-\zeta^2}t\right) \\ &= 1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\cos(\omega_n\sqrt{1-\zeta^2}t - \phi) \end{aligned}}$$

where $\phi = \tan^{-1}(\zeta/\sqrt{1-\zeta^2})$.

A plot of this response appears in Figure below for various values of ζ. We now see the relationship between the value of ζ and the type of response obtained: The lower the value of ζ, the more oscillatory the response. The natural frequency is a time-axis scale factor and does not affect the nature of the response other than to scale it in time.

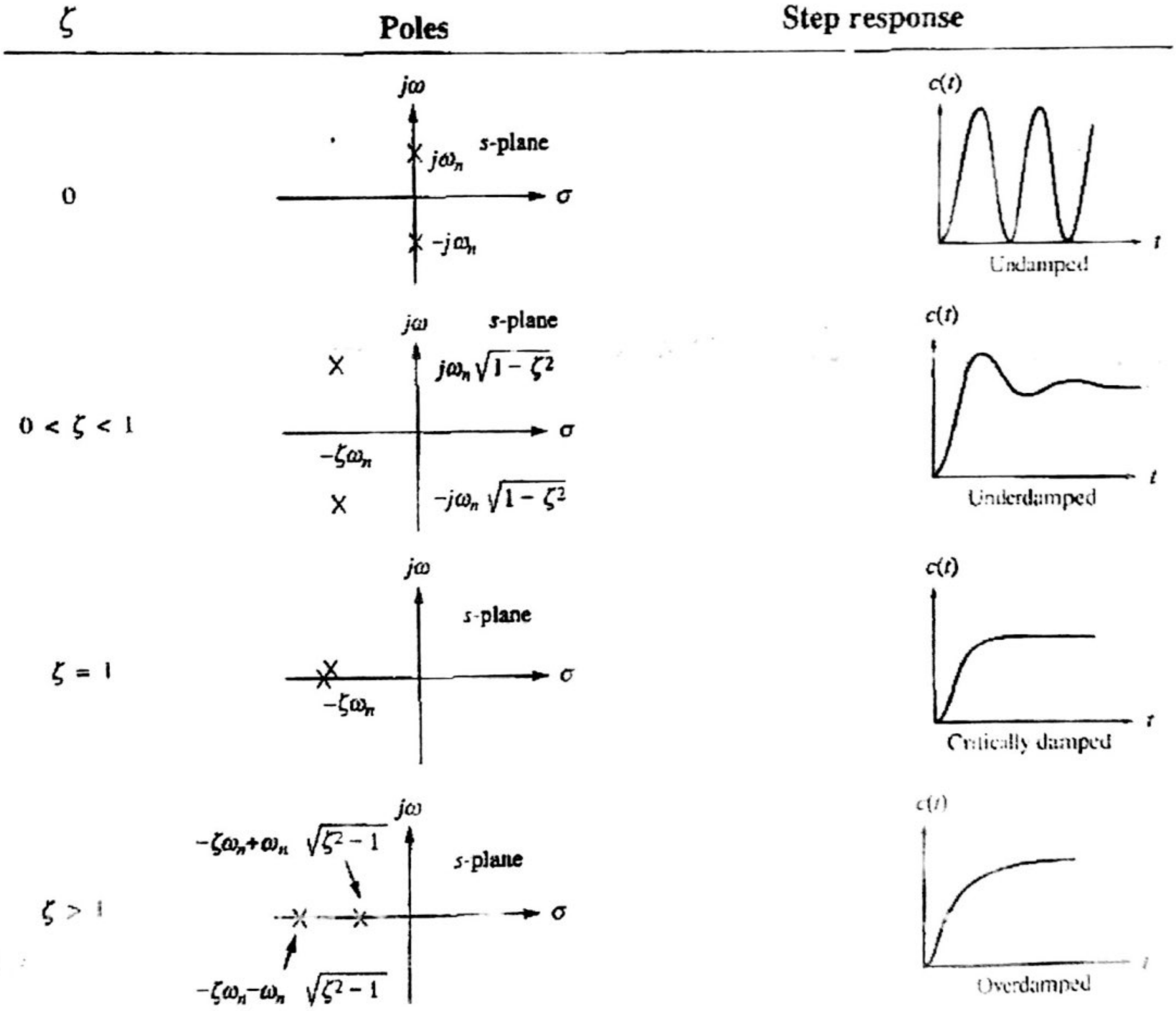


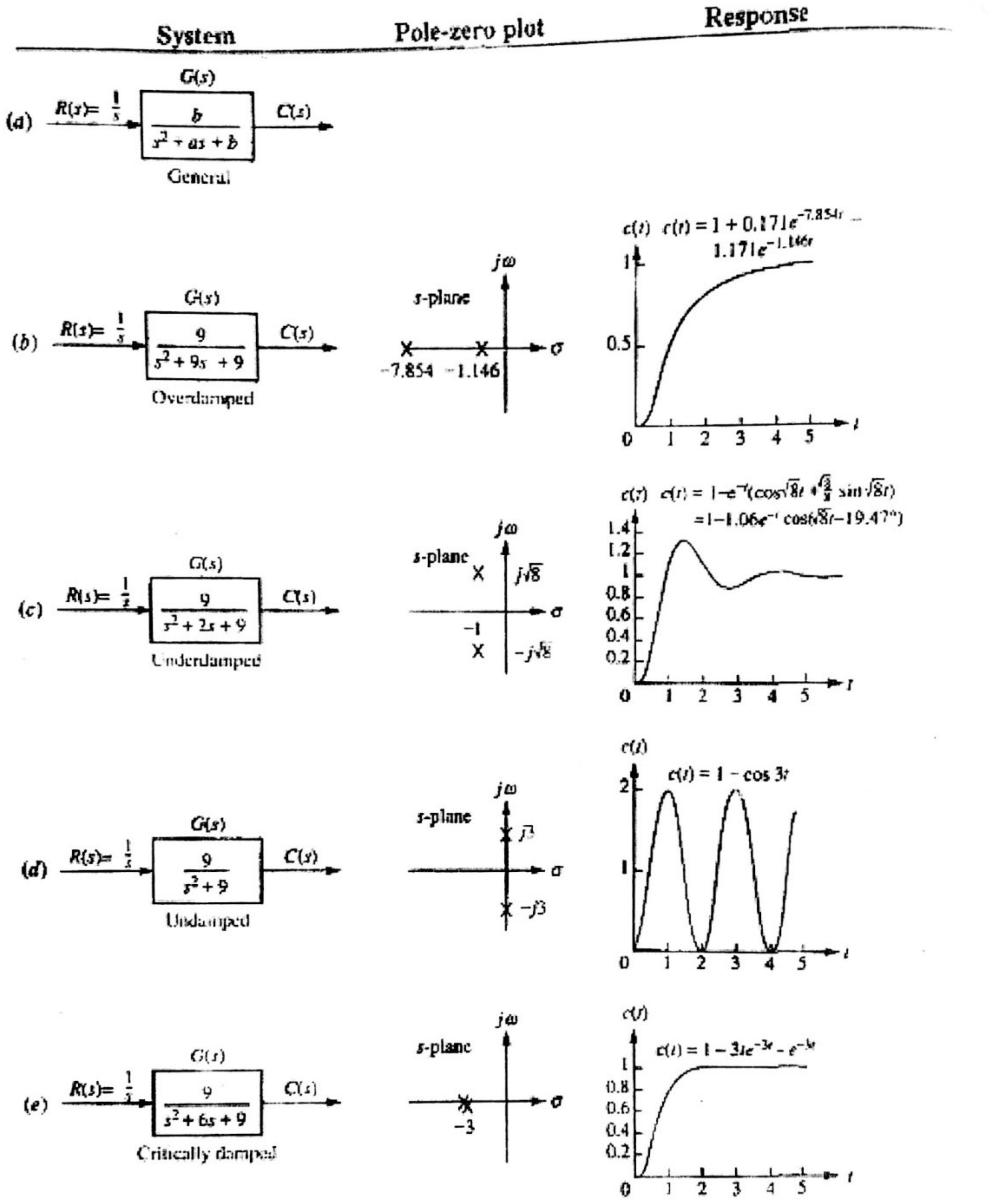
Depending on the value of (ζ) three distinct cases present as shown in table below

Three distinct cases of second order systems

Case	*Damping ratio value*	*Roots of characteristic equation*	*System specification*
1	$\xi > 1$	$s_1 \neq s_2$ and both are real.	Overdamped
2	$\xi = 1$	$s_1 = s_2$ and both are real.	Critically damped
3	$\xi < 1$	s_1 and s_2 are complex, one being the conjugate of the other.	Underdamped

We see that the various cases of second-order response are a function of ζ, they are summarized in figure below.




System
Pole-zero plot
Response
(a)
$R(s) = \frac{1}{s}$
$G(s)$
$\frac{b}{s^2 + as + b}$
$C(s)$
General
(b)
$R(s) = \frac{1}{s}$
$G(s)$
$\frac{9}{s^2 + 9s + 9}$
$C(s)$
Overdamped
$j\omega$
s-plane
-7.854
-1.146
σ
$c(t)$
$c(t) = 1 + 0.171e^{-7.854t} - 1.171e^{-1.146t}$
1
0.5
0
1
2
3
4
5
t
(c)
$R(s) = \frac{1}{s}$
$G(s)$
$\frac{9}{s^2 + 2s + 9}$
$C(s)$
Underdamped
$j\omega$
s-plane
$j\sqrt{8}$
-1
$-j\sqrt{8}$
σ
$c(t)$
$c(t) = 1 - e^{-t}(\cos\sqrt{8}t + \frac{\sqrt{8}}{8}\sin\sqrt{8}t)$
$= 1 - 1.06e^{-t}\cos(\sqrt{8}t - 19.47^\circ)$
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
t
(d)
$R(s) = \frac{1}{s}$
$G(s)$
$\frac{9}{s^2 + 9}$
$C(s)$
Undamped
$j\omega$
s-plane
$j3$
$-j3$
σ
$c(t)$
$c(t) = 1 - \cos 3t$
2
1
0
1
2
3
4
5
t
(e)
$R(s) = \frac{1}{s}$
$G(s)$
$\frac{9}{s^2 + 6s + 9}$
$C(s)$
Critically damped
$j\omega$
s-plane
-3
σ
$c(t)$
$c(t) = 1 - 3te^{-3t} - e^{-3t}$
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
t

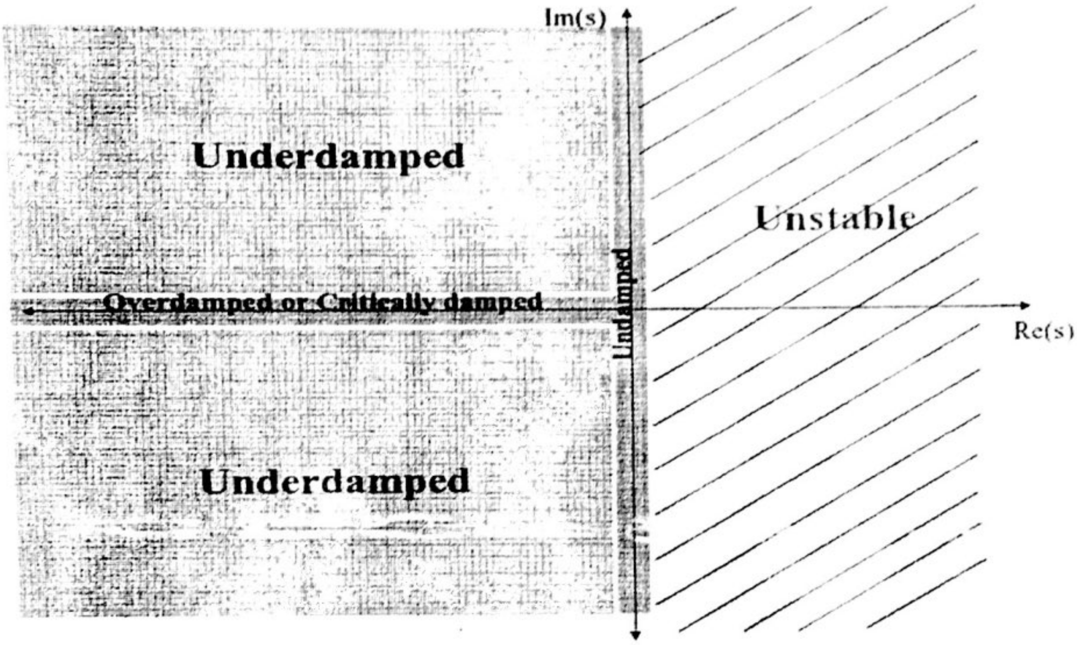
$0<\zeta<1$: $s_1, s_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ $(-\zeta\omega_n < 0)$ *underdamped*

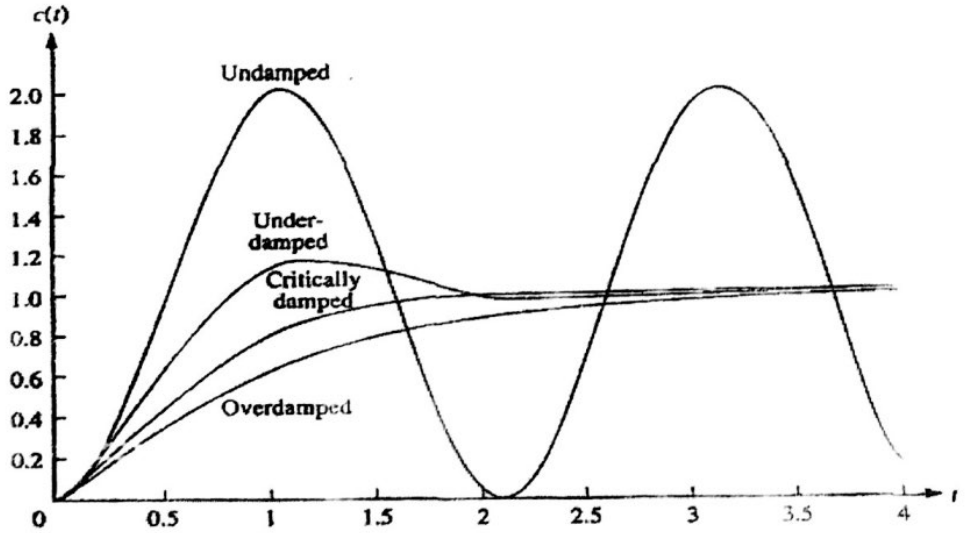
$\zeta = 1$: $s_1, s_2 = -\omega_n$ *critically damped*

$\zeta > 1$: $s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$ *overdamped*

$\zeta = 0$: $s_1, s_2 = \pm j\omega_n$ *undamped*

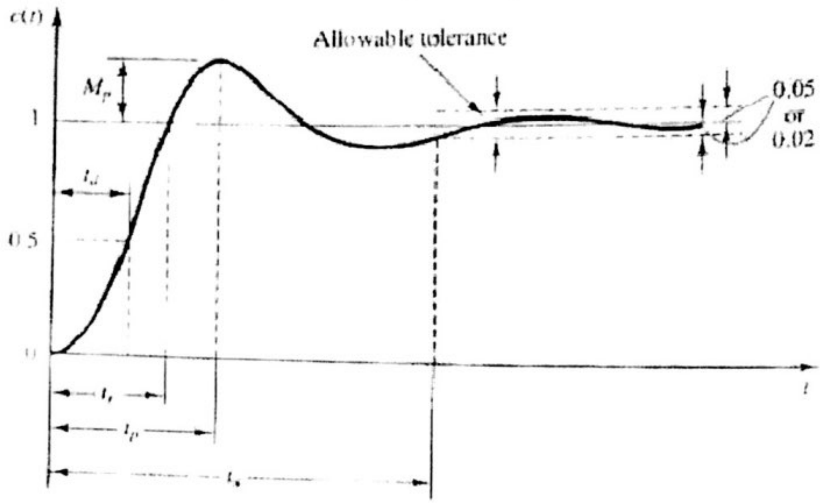
$\zeta < 0$: $s_1, s_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ $(-\zeta\omega_n > 0)$ *negatively damped*

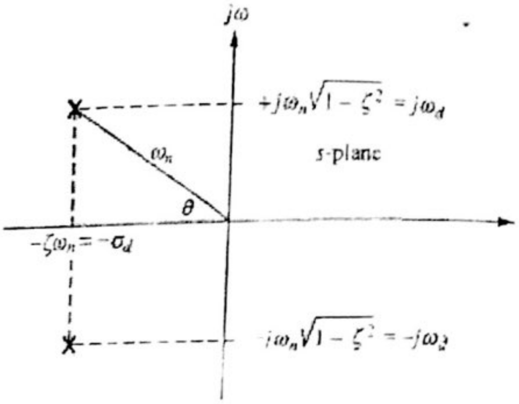




Step responses for second-order system damping cases

Definition of transient response specification




$j\omega$
$+j\omega_n\sqrt{1-\zeta^2} = j\omega_d$
s-plane
ω_n
θ
$-\zeta\omega_n = -\sigma_d$
$-j\omega_n\sqrt{1-\zeta^2} = -j\omega_d$

1. Delay time Td: The delay time is the time required for the response to reach half the final value the very first time. $T_d = (1+0.7\xi)/\omega_n \quad 0<\xi<1$

2. Rise time Tr: The time required for the waveform to go from 0.1 of the final value to 0.9 of the final value

$0<\xi<1$

$$T_r = \frac{\pi-\theta}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi-\tan^{-1}(\sqrt{1-\zeta^2}/\zeta)}{\omega_d} = \frac{\pi-\cos^{-1}(\zeta)}{\omega_d}\ , \quad T_r = \frac{2.16\zeta+0.6}{\omega_n}\ , T_r \approx \frac{0.8+2.5\xi}{\omega_n}$$

3. Peak time Tp: The peak time is the time required for the response to reach the first peak of the overshoot.

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega_d}$$

4. Maximum (percent) overshoot Mp: The maximum overshoot is the maximum peak value of the response curve measured from unit. If the final steady-state value of the response differs from unity, then it's common to use the maximum percent overshoot.

$$M_p\% = OS\% = e^{-(\zeta\pi/\sqrt{1-\xi^2})}.100$$

$$M_p\% = OS\% = \frac{c(tp)-c(\infty)}{c(\infty)}.100 = \frac{C_{max}-C_{final}}{C_{final}}.100$$

5. Settling time Ts: The time required for the transient's damped oscillations to reach and stay within ±2% of the steady-state value.

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{\sigma_d}$$

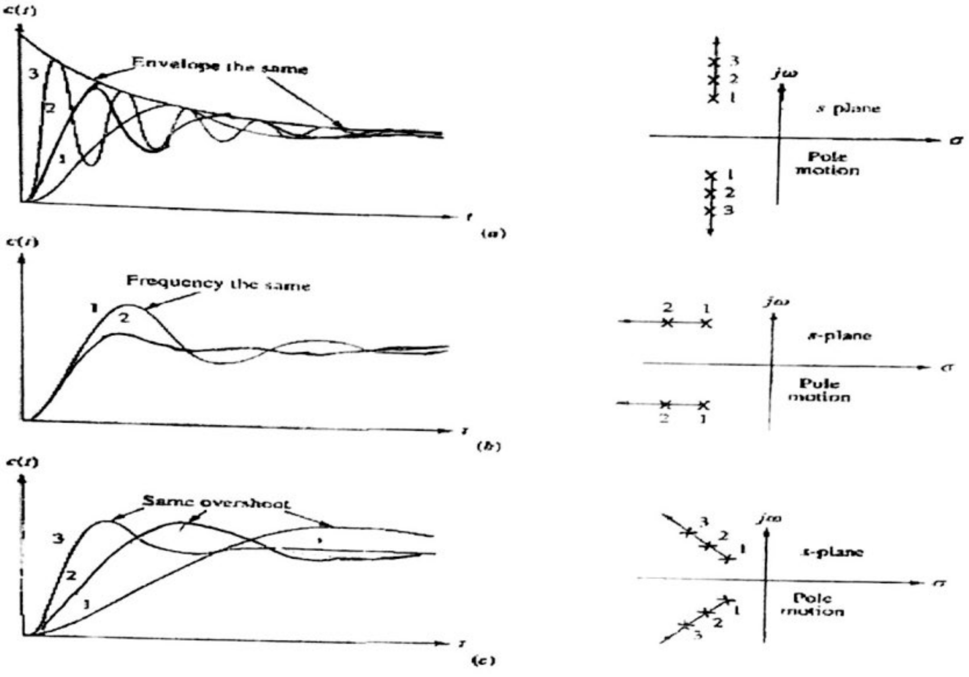
For varying within 5% of the final value, the setting time is $T_s \approx \frac{3}{\xi\omega_n}$

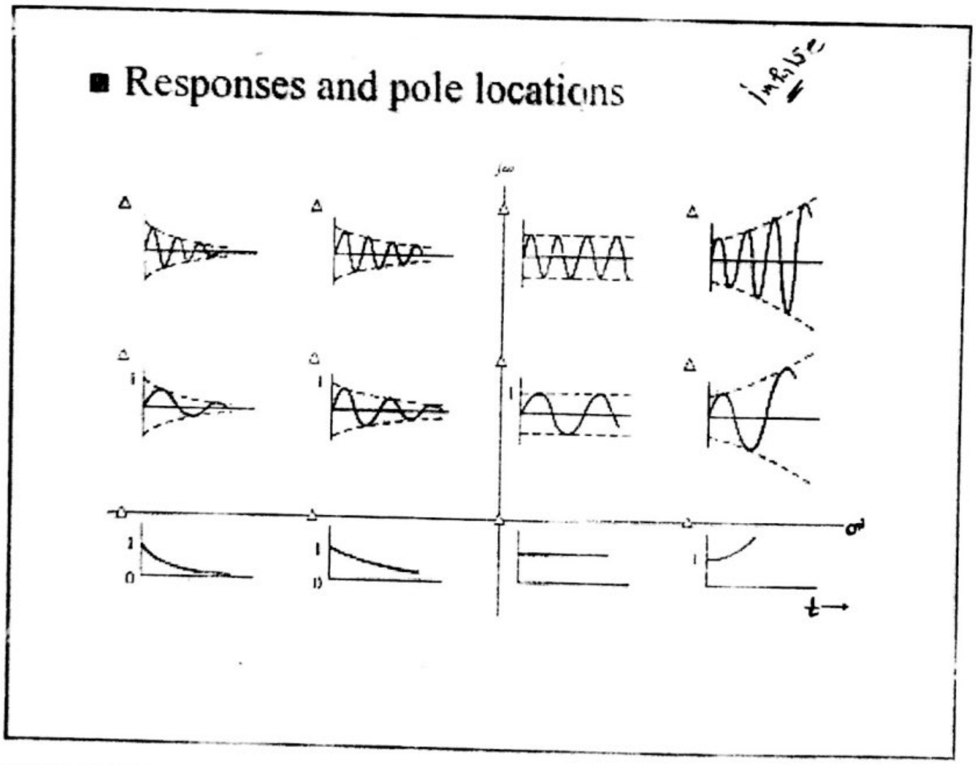
Step responses of second-order underdamped systems as poles move:

a. with constant real part
b. with constant imaginary part
c. with constant damping ratio

$T_P = \frac{\pi}{\omega_d}$

$T_s = \frac{4}{\sigma_d}$



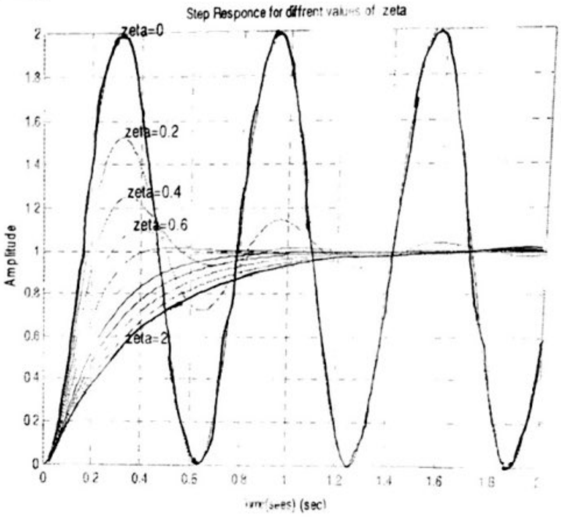

■ Responses and pole locations

Damping Ratio and Damping Factor

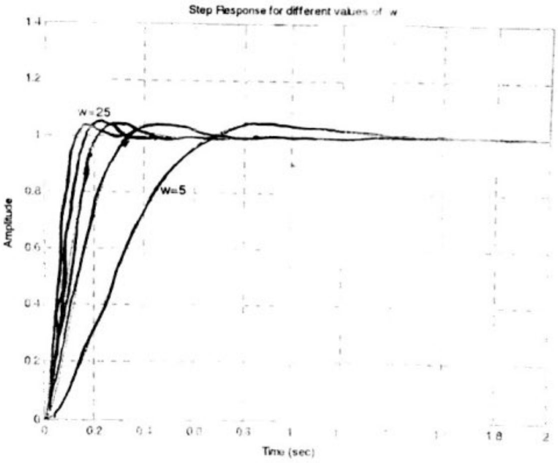
The effects of the system parameters ξ and ω_n„ on the step response y(t) of the prototype second-order system can be studied by referring to the roots of the characteristic equation.

The corresponding time responses for the following figure are obtained by the following sequence of MATLAB functions.

```
clc
clf
w=10;
for zeta=[ 0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2]
num = [w^2 ] ;
den= [1 2*zeta*w  w^2];
t=0:0.01:2;
step(num,den, t)
hold on;
disp('zeta and poles')
zeta
roots([1 2*zeta*w  w^2])
end
xlabel('Time(sees)')
ylabel('Amplitude')
title( 'Step Response for different values of
zeta' )
grid
```



```
clc
clf
zeta=0.7 ;
for w=[ 5 10 15 20 25 ]
num = [w^2 ] ;
den= [1 2*zeta*w  w^2];
t=0:0.01:2;
step(num,den, t)
hold on;
disp('zeta and poles')
w
roots([1 2*zeta*w  w^2])
end
xlabel('Time')
ylabel('Amplitude')
title( 'Step Response for different values of  w' )
grid
```



EX A step input is applied to a unity feedback system having the following open-loop transfer function:

$$G(s) = \frac{15}{s(s+3)}$$

Find (a) the closed-loop transfer function, (b) ω_n, (c) ξ and (d) ω_d.

Solution:

(a) Closed-loop transfer function $= \frac{G(s)}{1+G(s)} = \frac{15}{s(s+3)+15} = \frac{15}{s^2+3s+15}$

(b) Comparing the denominator of the closed-loop transfer function with that of a standard second order transfer function, we get

$$\omega_n^2 = 15$$

or

$$\omega_n = \sqrt{15} = 3.873 \text{ rad/s}$$

(c) From the same comparison, we also get

$$2\xi\omega_n = 3$$

or

$$\xi = \frac{3}{2\omega_n} = \frac{3}{2\sqrt{15}} = 0.387$$

(d)

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 3.571 \text{ rad/s}$$

EX A second order control system, having $\xi = 0.4$ and $\omega_n = 5$ rad/s, is subjected to a step input. Determine (a) closed-loop transfer function, (b) t_r, (c) t_p, (d) t_s and (e) M_p.

Solution:

(a) The standard closed-loop transfer function for a second order control system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Substituting the values of ξ and ω_n, we get

$$\frac{C(s)}{R(s)} = \frac{25}{s^2+4s+25}$$

(b)

$$t_r = \frac{\pi - \cos^{-1}\xi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi - \cos^{-1}0.4}{5\sqrt{1-0.16}} = 0.433 \text{ s}$$

(c)

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{5\sqrt{1-0.16}} = 0.686 \text{ s}$$

(d)

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{(0.4)(5)} = 2 \text{ s}$$

(e)

$$\%M_p = 100\exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right) = 100\exp\left(-\frac{0.4\pi}{\sqrt{1-0.16}}\right) = 25.4$$

EX In the closed-loop system shown in Figure determine the value of K and a such that the per cent peak overshoot due to the unit step input does not exceed 10%. Settling time should be equal to 1 s.

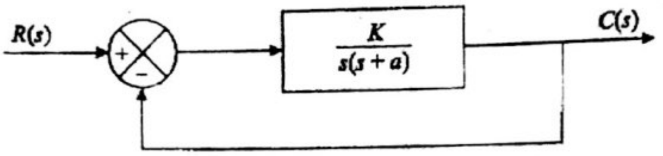


Fig. - Closed-loop system

Solution: The overall transfer function of the system is

$$T(s) = \frac{C(s)}{R(s)} = \frac{K}{s(s+a)+K}$$

$$= \frac{K}{s^2 + as + K}$$

Comparing the characteristic equation, $s^2 + as + K = 0$, with that of a standard second order system, we get

$$\omega_n = \sqrt{K}$$

$$\xi = \frac{a}{2\omega_n} = \frac{a}{2\sqrt{K}} \qquad \text{(i)}$$

Now, $$t_s = \frac{4}{\xi\omega_n} = 1\text{ s (given)}$$

$\therefore$ $$\xi\omega_n = 4$$

$$= \frac{a}{2} \text{ [using Eq. (i)]}$$

or $$a = 8 \qquad \text{(ii)}$$

The peak overshoot condition gives us

$$\exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right) = 0.1$$

$\Rightarrow$ $$\frac{\pi\xi}{\sqrt{1-\xi^2}} = 2.303 \qquad \text{(iii)}$$

Substituting the value of ξ from Eq. (i) in Eq. (iii), we get

$$\frac{\frac{\pi a}{2\sqrt{K}}}{\sqrt{1-\frac{a^2}{4K}}} = 2.303$$

$\Rightarrow$ $$\frac{\pi a}{\sqrt{4K - a^2}} = 2.303$$

Putting the value of a from Eq. (ii) in this equation, we get

$$\frac{\pi(8)}{\sqrt{4K - 64}} = 2.303$$

or $$\frac{64\pi^2}{2.303^2} = 4K - 64$$

or $$K = \frac{1}{4}\left[\frac{64\pi^2}{2.303^2} + 64\right] = 45.774$$

EX For the second-order systems that follow, find ξ, ω_n, Tr, Tp, Ts, OS.

a. $T(s) = \dfrac{16}{s^2 + 3s + 16}$ b. $T(s) = \dfrac{0.04}{s^2 + 0.02s + 0.04}$

(a)

$\omega_n^2 = 16$ r/s,

$2\zeta\omega_n = 3$. Therefore $\zeta = 0.375$,

$\omega_n = 4$

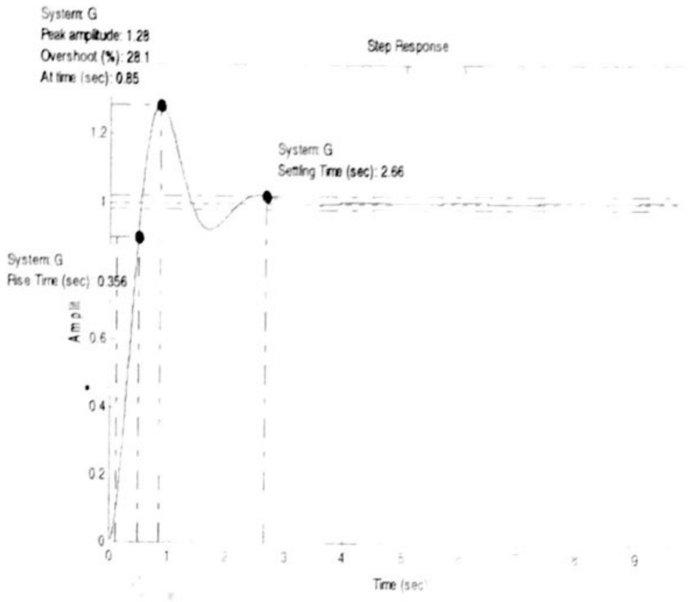
$$T_s = \frac{4}{\zeta\omega_n} = 2.667 \text{ s}$$

$$Tp = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.8472 \text{ s}$$

$$\%OS = e^{-\zeta\pi / \sqrt{1-\zeta^2}} \times 100 = 28$$

$$T_r = \frac{2.16\zeta + 0.6}{\omega_n} = \frac{2.16 * 0.375 + 0.6}{4}$$

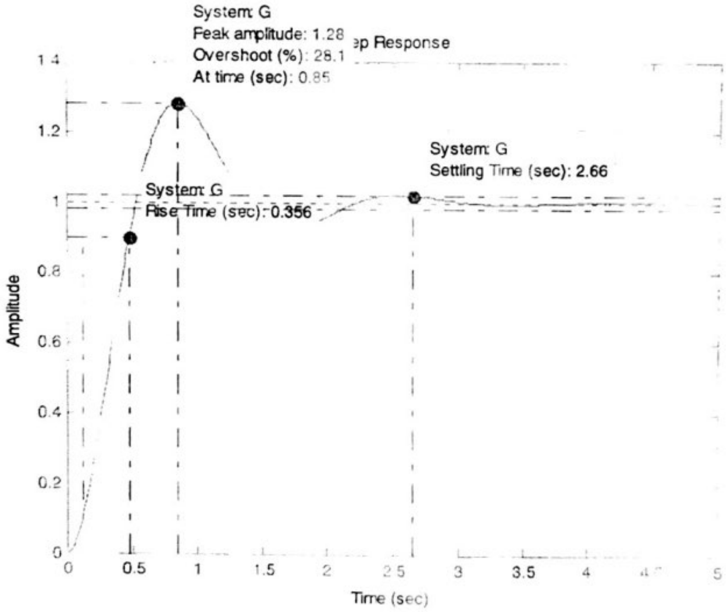
$$T_r == 0.35$$



```
clf
num=16;
t=0:0.01:10;
den=[1 3 16];
G=tf(num,den);
omegan=sqrt(den(3));
zeta=den(2)/(2*omegan);
Ts=4/(zeta*omegan);
Tp=pi/(omegan*sqrt(1-zeta^2));
OS=exp(-zeta*pi/sqrt(1-zeta^2))*100;
Tra=(1.76*zetaa^3 - 0.417*zetaa^2 + 1.039*zetaa + 1)/omegana
[y,t] = step(G,t);
step(G,t)
disp(' omega            zeta      Tr        Tp         Ts         OS%')
[omegan    zeta    Tr    Tp  Ts    OS]
```

```
      omega      zeta       Tr        Tp         Ts        OS%

      4.0000     0.3750     0.3600    0.8472     2.6667    28.0597
```

```
% another solution
clc
num=16;
t=0:0.01:5;
den=[1 3 16];
G=tf(num,den);
omegan=sqrt(den(3));
zeta=den(2)/(2*omegan);
[y,t] = step(G,t);
y1= find(y>=0.1);
y9= find(y>=0.9);
ys= find(y>0.98 & y<1.02);
yd= find(y>=0.5);
n= max(y)
yp= find(y>=n);
Td= t(yd(1))                 % time delay
Tr=t(y9(1))-t(y1(1))         % rise time
Tp=t(yp(1))                  % peak time
Ts=t(ys(45))                 % settling time
os = (max(y) -y(end)/y(end))*100     % overshoot
step(G,t)

disp(' omega    zeta     Td     Tr  Tp     Ts      OS%')
[    omegan     zeta     Td      Tr     Tp   Ts    OS]
```



```
omega     zeta      Td      Tr      Tp      Ts      OS%

4.00      0.375     0.31    0.36    0.85    2.67    28.059
```

System Response with additional Poles

- If a system has more than two poles or has zeros, we cannot use the formulae to calculate the **performance specifications** that we derived.

- Under certain conditions, a system with more than two poles or with zeros can be approximated as a second-order system that has just two **complex dominant poles**.

- Once we justify this approximation, the formulas for percent overshoot, settling time, and peak time can be applied to these higher-order systems.

- Consider a three-pole system with complex poles and a third pole on the real axis. Assuming that the complex poles are at $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ and the real pole is at α_r. The step response of the system in time domain.

$$c(t) = Au(t) + e^{-\zeta\omega_n t}(B\cos\omega_d t + C\sin\omega_d t) + De^{-\alpha_r t}$$

- If $|\alpha_r| \gg |\zeta\omega_n|$ the pure exponential will die out much more rapidly than the second-order underdamped step response. If the pure exponential term decays to an insignificant value at the time of the first overshoot, such parameters as percent overshoot, settling time, and peak time will be generated by the second-order underdamped step response component. Thus, the total response will approach that of a pure second-order system.

- We can show, through a partial fraction expansion, that the residue of the third pole, in a three-pole system with dominant second-order poles and no zeros, will actually decrease in magnitude as the third pole is moved farther into the left half-plane.

Ex

```
num=5;
den=poly([ -1 -2 -20])
[r,p,k]= residue(num,den)

r =                                   p =
   0.0146                                    -20
  -0.2778                                     -2
   0.2632                                     -1
```

- If the real pole is five times farther to the left than the dominant poles, $|\alpha_r| > 5|\zeta\omega_n|$ we assume that the system is represented by its dominant second-order pair of poles. The step response of the system in time domain.

$$c(t) = Au(t) + e^{-\zeta\omega_n t}(B\cos\omega_d t + C\sin\omega_d t)$$

EX:

Determine the validity of a second-order approximation for each of these two transfer functions:

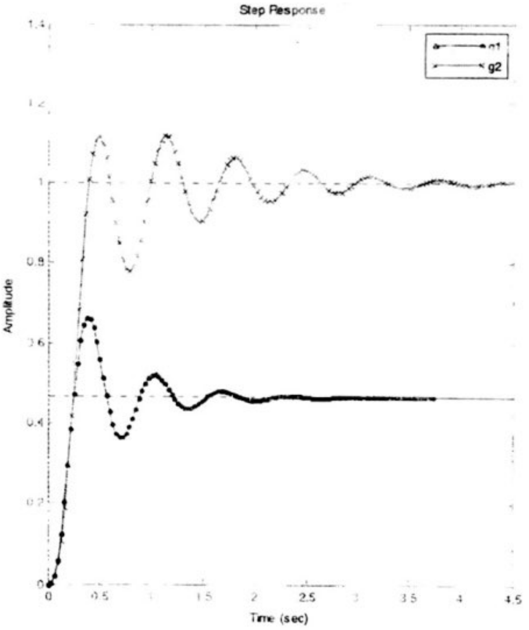
a. $G(s) = \dfrac{700}{(s+15)(s^2+4s+100)}$ b. $G(s) = \dfrac{360}{(s+4)(s^2+2s+90)}$

The second-order approximation is valid, since the dominant poles have a real part of –2 and the higher-order pole is at –15, i.e. more than five-times further.

b. The second-order approximation is not valid, since the dominant poles have a real part of –1 and the higher-order pole is at –4, i.e. not more than five-times further.

```
num1= 700;
den1=conv([1 15],[1 4 100]);
poles1=roots(den1)
g1= tf(num1,den1);

num2= 360;
den2=conv([1 4],[1 2 90]);
poles2=roots(den2)
g2= tf(num2,den2);

step(g1,g2)
```

```
poles1 =

 -15.0000
  -2.0000 + 9.7980i
  -2.0000 - 9.7980i


poles2 =

 -1.0000 + 9.4340i
 -1.0000 - 9.4340i
 -4.0000
```

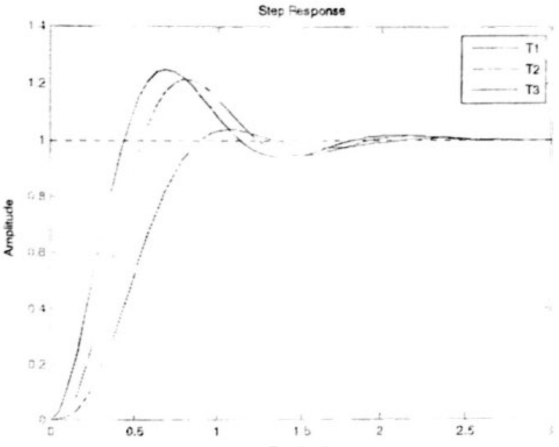


Ex Find the step response of each of the following transfer functions using
a- MATLAB
b- Simulink
c- GUI

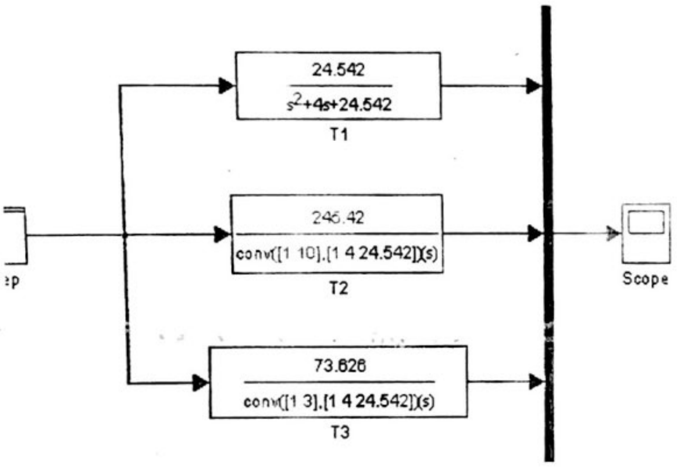
$$T_1(s)=\frac{24.542}{s^2+4s+24.542} \qquad T_2(s)=\frac{245.42}{(s+10)(s^2+4s+24.542)} \qquad T_3(s)=\frac{73.626}{(s+3)(s^2+4s+24.542)}$$

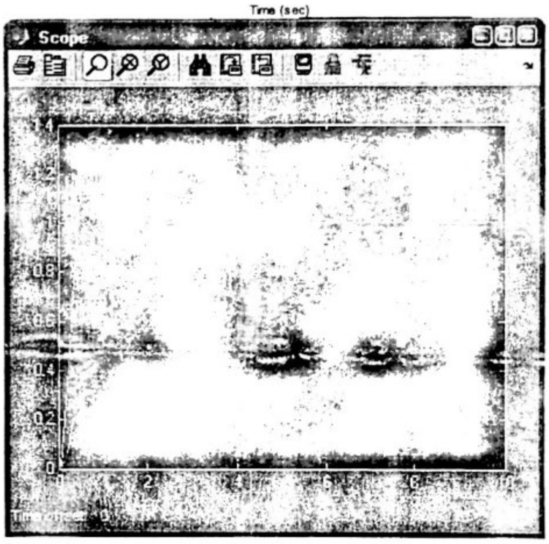
a- MATLAB

```
num1 =[24.542];
den1=[ 1 4 24.542]
T1= tf(num1,den1)
num2 =[245.42];
den2=conv([1 10],[1 4 24.542])
T2= tf(num2,den2)
num3 =[73.626];
den3=conv([1 3],[1 4 24.542])
T3= tf(num3,den3)
step(T1,T2,T3)
legend('T1','T2','T3')
```

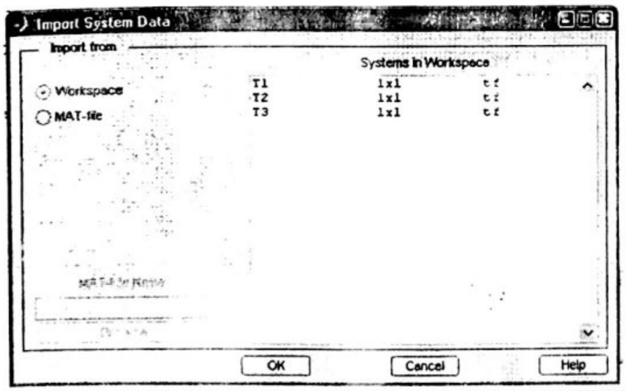


b- simulink



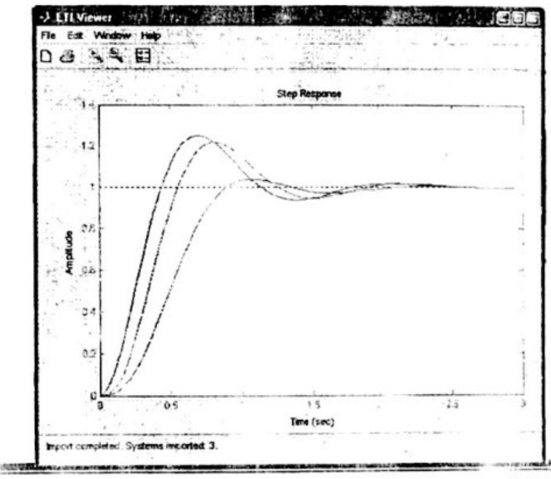


c- GUI : Another method to obtain systems response is though the use ltiview
>> ltiview
1- Select file 2- select import from file menu



T1	1x1	tf
T2	1x1	tf
T3	1x1	tf

Ex



Import completed. Systems imported: 3.

System Response with Zeros

The transfer function for a second-order system can be written in one of the two general forms (depending on whether the system has a zero or not).

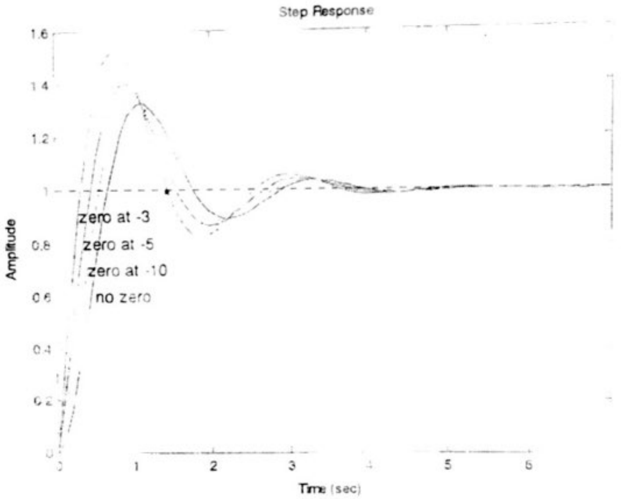
$$Case1 : G1(s) = \frac{K}{s^2 + bs + c} \quad , \quad Case2 : G2(s) = \frac{K(s+a)}{s^2 + bs + c} = sG1(s) + aG1(s)$$

Starting with a two-pole system with poles at (-1 ±j2.828), we consecutively add zeros at -3, -5, and -10.

$$T = \frac{9}{s^2 + 2s + 9}, \quad T_a = \frac{3(s+3)}{s^2 + 2s + 9}, \quad T_b = \frac{1.8(s+5)}{s^2 + 2s + 9}, \quad T_c = \frac{0.9(s+10)}{s^2 + 2s + 9}$$

The results, normalized to the steady-state value, are plotted in Figure below.

```
deng=[1 2 9];
Ta=tf ([1 3]*9/3,deng);
Tb=tf([1 5] *9/5,deng);
Tc=tf ([1 10] * 9/10, deng);
T=tf (9,deng);
step(T,Ta,Tb,Tc)
text (0.5, 0.6, 'no zero')
text(0.4,0.7,'zero at -10')
text(0.35, 0.8, 'zero at -5')
text (0.3,0.9,'zero at -3')
```



- The motion of Case 2 systems is complicated somewhat over that of Case 1 systems by the presence of the zero (s+a) in the transfer function.

- The zero has ***little effect on the settling time*** of the system, but ***can significantly affect the overshoot***.

- How much effect the zero has on overshoot depends on where it is (along the real axis) compared to the poles of the system.

o ***Zero is far to the left of the poles***: it has little affect on the motion of the system. This makes the motion much like that of a Case 1 system.

o ***Zero located near or inside the poles***: it will significantly affect the overshoot of the system.

Evaluating Pole-Zero Cancellation Using Residues

Assume a three pole system with a zero as shown

$$T(s) = \frac{k(s+z)}{(s+p_3)(s^2+\ as+b)}$$

If the pole term, $(s+p_3)$, and the zero term (s + z), cancel out, we are left with

$$T(s) = \frac{K\cancel{(s+z)}}{\cancel{(s+p_3)}\ (s^2+as+b)}$$

as a second-order transfer function. From another perspective, if the zero at - z is very close to the pole at –p3 , then a partial-fraction expansion of T(s) will show that the residue of the exponential decay is much smaller than the amplitude of the second-order response. Let us look at an example.

Ex

Determine the validity of a second-order step-response approxima-tion for each transfer function shown below, find the approximate response.

$$C_1(s) = \frac{26.25(s+4)}{s(s+3.5)(s+5)(s+6)} \qquad C_2(s) = \frac{26.25(s+4)}{s(s+4.01)(s+5)(s+6)}$$

The partial-fraction expansion of C1(s)

$$C_1(s) = \frac{1}{s} - \frac{3.5}{s+5} + \frac{3.5}{s+6} - \frac{1}{s+3.5}$$

The residue of the pole at -3.5, which is closest to the zero at -4, is equal to 1 and is not negligible compared to the other residues. **Thus, a second-order step response approximation cannot be made for C1(s).**

The partial-fraction expansion of C2(s)

$$C_2(s) = \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} + \frac{0.033}{s+4.01}$$

The residue of the pole at -4.01, which is closest to the zero at -4, is equal to 0.033, about two orders of magnitude below any of the other residues. Hence, we make a second-order approximation by neglecting the response generated by the pole at -4.01:

$$C_2(s) \approx \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6}$$

and the response $c_2(t)$ is approximately

$$c_2(t) \approx 0.87 - 5.3e^{-5t} + 4.4e^{-6t}$$

Laplace Transform Solution of State Equations

Consider the state equation

$$\dot{x} = \mathrm{A}x + \mathrm{B}u$$

and the output equation

$$y = \mathrm{C}x + \mathrm{D}u$$

Taking the Laplace transform of both sides of the state equation yields

$$sx(s) - x(0) = AX(s) + BU(s)$$

$$sX(s) - x(0) = \underset{x}{\underline{A(s)}} + BU(s)$$

$$(sI - A)\, X(s) = x(0) + BU(s)$$

$$\mathbf{X(s) = (sI - A)^{-1}\, [x(0) + BU(s)]}$$

Taking the Laplace transform of the output equation yields

$$Y(s) = CX(s) + DU(s)$$

Note : The eigenvalues of the system matrix A = Roots of det(sI-A)=0.

Ex:

Given the system represented in state space

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 2 \\ -3 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-t} \qquad \mathbf{x}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad y = [1 \quad 3]\mathbf{x}$$

do the following:

a. Solve for y(t) using state-space and Laplace transform techniques.

b. Find the eigenvalues and the system poles.

a. Since $s\mathbf{I} - \mathbf{A} = \begin{bmatrix} s & -2 \\ 3 & s+5 \end{bmatrix}$, $(s\mathbf{I} - \mathbf{A})^{-1} = \dfrac{1}{s^2 + 5s + 6}\begin{bmatrix} s+5 & 2 \\ -3 & s \end{bmatrix}$. Also.

$$\mathbf{BU}(s) = \begin{bmatrix} 0 \\ 1/(s+1) \end{bmatrix}.$$

The state vector is $\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}[\mathbf{x}(0) + \mathbf{BU}(s)] = \dfrac{1}{(s+1)(s+2)(s+3)}\begin{bmatrix} 2(s^2 + 7s + 7) \\ s^2 - 4s - 6 \end{bmatrix}$.

The output is $Y(s) = [1 \quad 3]\mathbf{X}(s) = \dfrac{5s^2 + 2s - 4}{(s+1)(s+2)(s+3)} = -\dfrac{0.5}{s+1} - \dfrac{12}{s+2} + \dfrac{17.5}{s+3}$.

Taking the inverse Laplace transform yields $y(t) = -0.5e^{-t} - 12e^{-2t} + 17.5e^{-3t}$.

b. The eigenvalues are given by the roots of $|s\mathbf{I} - \mathbf{A}| = s^2 + 5s + 6$, or -2 and -3.

MATLAB code

```
clc
syms s
A=[0 2;-3 -5];
B=[ 0;1];
C=[ 1 3];
X0=[2;1];
U=1/(s+1) ;  %u(t)=exp(-t)
I=eye(2);
X=((s*I-A)^-1)*(X0+B*U);
x1=ilaplace(X(1));
x2=ilaplace(X(2));
y=x1+3*x2;
pretty(y)
'Eigen valus'
E=eig(A)
```

```
y(t)=


 35 exp(-3 t)                    exp(-t)
 ------------ - 12 exp(-2 t) - -------
      2                            2



Eigen valus


E =

   -2.0000
   -3.0000
```

Time Domain Solution of State Equations

The solution in the time domain is given directly by

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)d\tau$$

$$= \mathbf{\Phi}(t)\mathbf{x}(0) + \int_0^t \Phi(t-\tau)\mathbf{B}\mathbf{u}(\tau)d\tau$$

$\Phi(0) = I$

$\dot{\Phi}(0) = A$

$\emptyset(t)$: **state transition matrix,** $\emptyset(t) = e^{At}$

$$\mathbf{\Phi}(t) = \mathscr{L}^{-1}[(s\mathbf{I}-\mathbf{A})^{-1}] = \mathscr{L}^{-1}\left[\frac{\operatorname{adj}(s\mathbf{I}-\mathbf{A})}{\det(s\mathbf{I}-\mathbf{A})}\right]$$

EX

For the following state equation and initial state vector. Find the state-transition matrix and then solve for x(t). Where u(t) is a unit step.

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t)$$

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

We use the fact that $\mathbf{\Phi}(t)$ is the inverse Laplace transform of $(s\mathbf{I}-\mathbf{A})^{-1}$. Thus, first find $(s\mathbf{I}-\mathbf{A})$ as

$$(s\mathbf{I}-\mathbf{A})=\begin{bmatrix} s & -1 \\ 8 & (s+6) \end{bmatrix}$$

from which

$$(s\mathbf{I}-\mathbf{A})^{-1}=\frac{\begin{bmatrix} s+6 & 1 \\ -8 & s \end{bmatrix}}{s^2+6s+8}=\begin{bmatrix} \dfrac{s+6}{s^2+6s+8} & \dfrac{1}{s^2+6s+8} \\ \dfrac{-8}{s^2+6s+8} & \dfrac{s}{s^2+6s+8} \end{bmatrix}$$

Expanding each term in the matrix on the right by partial fractions yields

$$(s\mathbf{I}-\mathbf{A})^{-1}=\begin{bmatrix} \left(\dfrac{2}{s+2}-\dfrac{1}{s+4}\right) & \left(\dfrac{1/2}{s+2}-\dfrac{1/2}{s+4}\right) \\ \left(\dfrac{-4}{s+2}+\dfrac{4}{s+4}\right) & \left(\dfrac{-1}{s+2}+\dfrac{2}{s+4}\right) \end{bmatrix}$$

Finally, taking the inverse Laplace transform of each term, we obtain

$$\mathbf{\Phi}(t)=\begin{bmatrix} (2e^{-2t}-e^{-4t}) & \left(\frac{1}{2}e^{-2t}-\frac{1}{2}e^{-4t}\right) \\ (-4e^{-2t}+4e^{-4t}) & (-e^{-2t}+2e^{-4t}) \end{bmatrix}$$

$$\mathbf{\Phi}(t-\tau)\mathbf{B}=\begin{bmatrix} \left(\frac{1}{2}e^{-2(t-\tau)}-\frac{1}{2}e^{-4(t-\tau)}\right) \\ \left(-e^{-2(t-\tau)}+2e^{-4(t-\tau)}\right) \end{bmatrix}$$

$$\mathbf{\Phi}(t)\mathbf{x}(0)=\begin{bmatrix} (2e^{-2t}-e^{-4t}) \\ (-4e^{-2t}+4e^{-4t}) \end{bmatrix}$$

$$\int_0^t \mathbf{\Phi}(t-\tau)\mathbf{B}\mathbf{u}(\tau)d\tau=\begin{bmatrix} \frac{1}{2}e^{-2t}\int_0^t e^{2\tau}d\tau-\frac{1}{2}e^{-4t}\int_0^t e^{4\tau}d\tau \\ -e^{-2t}\int_0^t e^{2\tau}d\tau+2e^{-4t}\int_0^t e^{4\tau}d\tau \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1}{8}-\frac{1}{4}e^{-2t}+\frac{1}{8}e^{-4t} \\ \frac{1}{2}e^{-2t}-\frac{1}{2}e^{-4t} \end{bmatrix}$$

$$\mathbf{x}(t)=\mathbf{\Phi}(t)\mathbf{x}(0)+\int_0^t \Phi(t-\tau)\mathbf{B}\mathbf{u}(\tau)d\tau=\begin{bmatrix} \frac{1}{8}+\frac{7}{4}e^{-2t}-\frac{7}{8}e^{-4t} \\ -\frac{7}{2}e^{-2t}+\frac{7}{2}e^{-4t} \end{bmatrix}$$

MATLAB code

```
clc
syms s  t   tau
A=[0 1;-8 -6];
B=[ 0;1];
X0=[1;0];
U=1 ;  %u(t)=step function
I=eye(2);
E=((s*I-A)^-1);
Fi= ilaplace(E);
pretty(Fi)
Fitmtau=subs(Fi,t,t-tau);
'Fi(t-ta)'
pretty(Fitmtau)
x= Fi*X0+int(Fitmtau*B*1,tau,0,t);
'x(t)'
pretty(x)
```

```
                                                            -+
        +-                                exp(-2 t)   exp(-4 t)  |
        |                                                  |
        |   2 exp(-2 t) - exp(-4 t),  --------- - ---------  |
  Fi =  |                                   2           2      |
        |                                                  |
        |   4 exp(-4 t) - 4 exp(-2 t), 2 exp(-4 t) - exp(-2 t) |
        +-                                                -+
```

```
                                                                            -+
           +                                     exp(2 tau - 2 t)   exp(4 tau - 4 t)   |
           |                                                                           |
           |   2 exp(2 tau - 2 t) - exp(4 tau - 4 t),  ---------------- - ----------------  |
Fi(t-ta) = |                                                 2                  2         |
           |                                                                           |
           |  4 exp(4 tau - 4 t) - 4 exp(2 tau - 2 t), 2 exp(4 tau - 4 t) - exp(2 tau - 2 t) |
           +-                                                                         -+
```

```
           +-                                   -+
           |  7 exp(-2 t)    7 exp(-4 t)          |
           |  ----------- - ----------- + 1/8    |
           |       4              8               |
  x(t)  =  |                                      |
           |     7 exp(-4 t)    7 exp(-2 t)       |
           |     ----------- - -----------        |
           |          2              2            |
           +-                                   -+
```

Ex

Determine the validity of a second-order step-response approxima-tion for each transfer function shown below.

a. $G(s) = \dfrac{185.71(s+7)}{(s+6.5)(s+10)(s+20)}$ **b.** $G(s) = \dfrac{197.14(s+7)}{(s+6.9)(s+10)(s+20)}$

a. Expanding $G(s)$ by partial fractions yields $G(s) = \dfrac{1}{s} + \dfrac{0.8942}{s+20} - \dfrac{1.5918}{s+10} - \dfrac{0.3023}{s+6.5}$.

But -0.3023 is not an order of magnitude less than residues of second-order terms (term 2 and 3). Therefore, a second-order approximation is not valid.

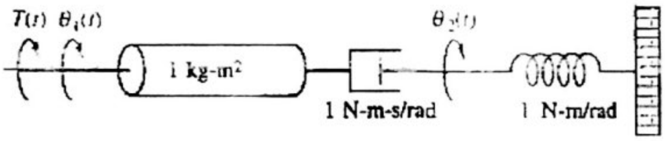
b. Expanding $G(s)$ by partial fractions yields $G(s) = \dfrac{1}{s} + \dfrac{0.9782}{s+20} - \dfrac{1.9078}{s+10} - \dfrac{0.0704}{s+6.5}$.

But 0.0704 is an order of magnitude less than residues of second-order terms (term 2 and 3). Therefore, a second-order approximation is valid.

Ex

For the system shown in Figure below, a step torque is applied at $\theta_1(t)$. Find

a. The transfer function, $G(s) = \theta_2(t) / T(t)$ b. The percent overshoot, settling time, and peak time.



Writing the loop equations,

$$(s^2+s)\,\theta_1(s) - s\,\theta_2(s) = T(s)$$

$$-s\,\theta_1(s) + (s+1)\,\theta_2(s) = 0$$

Solving for $\theta_2(s)$.

$$\theta_2(s) = \frac{\begin{vmatrix} s^2+s & T(s) \\ -s & 0 \end{vmatrix}}{\begin{vmatrix} s^2+s & -s \\ -s & s+1 \end{vmatrix}} = \frac{T(s)}{s^2+s+1}$$

Forming the transfer function.

$$\frac{\theta_2(s)}{T(s)} = \frac{1}{s^2+s+1}$$

Thus $\omega_n = 1$, $2\zeta\omega_n = 1$. Thus, $\zeta = 0.5$. %OS = 16.3%. $T_s = 8$ seconds $T_p = 3.63$ seconds.

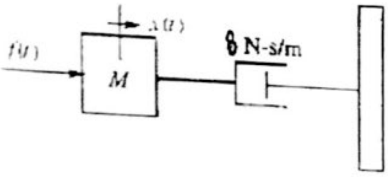
Ex: For the system shown in figure below, (a) find an equation that relates settling time of the velocity of the mass to M; (b) find an equation that relates rise time of the velocity of the mass to M.

Writing the equation of motion.

Thus, the transfer function is,

$$(Ms^2 + 8s)X(s) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + 8s}$$



Differentiating to yield the transfer function in terms of velocity,

$$\frac{sX(s)}{F(s)} = \frac{1}{Ms + 8} = \frac{1/M}{s + \frac{8}{M}}$$

Thus, the settling time, T_s, and the rise time, T_r, are given by

$$T_s = \frac{4}{8/M} = \frac{1}{2}M; \quad T_r = \frac{2.2}{8/M} = 0.275M$$

Ex For each of the transfer functions shown below, find the locations of the poles and zeros, plot them on the s.-plane, and then write an expression for the general form of the step response without solving for the inverse Laplace transform. State the nature of each response (overdamped, underdamped, and so on).

a. $T(s) = \dfrac{2}{s+2}$

b. $T(s) = \dfrac{5}{(s+3)(s+6)}$

c. $T(s) = \dfrac{10(s+7)}{(s+10)(s+20)}$

d. $T(s) = \dfrac{20}{s^2 + 6s + 144}$

e. $T(s) = \dfrac{s+2}{s^2+9}$

f. $T(s) = \dfrac{(s+5)}{(s+10)^2}$

a. Pole: -2; $c(t) = A + Be^{-2t}$; first-order response.

b. Poles: -3, -6; $c(t) = A + Be^{-3t} + Ce^{-6t}$; overdamped response.

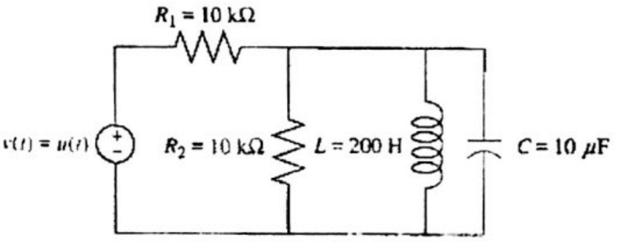
c. Poles: -10, -20; Zero: -7; $c(t) = A + Be^{-10t} + Ce^{-20t}$; overdamped response.

d. Poles: $(-3+j3\sqrt{15})$, $(-3-j3\sqrt{15})$; $c(t) = A + Be^{-3t}\cos(3\sqrt{15}\,t + \phi)$; underdamped.

e. Poles: j3, -j3; Zero: -2; $c(t) = A + B\cos(3t + \phi)$; undamped.

f. Poles: -10, -10; Zero: -5; $c(t) = A + Be^{-10t} + Cte^{-10t}$; critically damped.

Ex Write the general form of the capacitor voltage for the electrical network shown in Figure below.



Writing the node equation at the capacitor,

$$V_C(s)\left(\frac{1}{R_2} + \frac{1}{Ls} + Cs\right) + \frac{V_C(s) - V(s)}{R_1} = 0.$$

Hence, $$\frac{V_C(s)}{V(s)} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{Ls} + Cs} = \frac{10s}{s^2+20s+500}$$

The step response is $\frac{10}{s^2+20s+500}$

The poles are at

$-10 \pm j20$. Therefore, $v_C(t) = Ae^{-10t}\cos(20t + \phi)$.

HW 5

Problem 1

A RC circuit has the following transfer function: $\frac{Y(s)}{R(s)} = \frac{2}{10s+4}$ For a step input $r(t) = 2V$, what is the time taken for the output of the RC circuit to reach 95% of its steady-state response?

Problem 2

A system has transfer function: $\frac{Y(s)}{R(s)} = \frac{50}{s+50}$

Find the time constant, T, the settling time, T_s, and the rise time, T_r for a unit-step input.

Problem 3

Consider the unit-step response of a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{1}{s(s+1)}$$

Obtain the rise time, peak time, maximum overshoot, and settling time.

Problem 4

A second-order system is underdamped with a damping ratio of 0.4 and a natural frequency of 10Hz. Find:

a) the transfer function
b) the time response when it is subjected to a unit-step input
c) the percentage overshoot with such an input
d) the rise time

Problem 5

Given the transfer function:

$$G(s) = \frac{100}{s^2 + 15s + 100}$$

find T_p, PO, T_s and T_r.

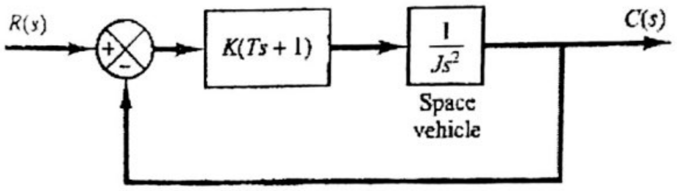
Problem 6

Obtain the unit-impulse response and the unit-step response of a unity-feedback system whose open-loop transfer function is

$$G(s) = \frac{2s+1}{s^2}$$

Problem 7

The following block diagram of a space-vehicle attitude control system. Assume the time constant T to be 3 sec, and the ratio K/J to be (2/9) rad^2/sec. Find the daming ratio of the system.



Problem 8

Obtain the unit-step response, unit-ramp response, and unit-impulse response of the following system and repeat your solution using MATLAB.

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + 2s + 10}$$

Problem 9

Obtain the unit-step response, unit-ramp response, and unit-impulse response of the following system and repeat your solution using MATLAB.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where u is the input and y is the output.

Problem 10

Given the following state-space representation of a system, find Y(s):

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 2 \\ -4 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \sin 3t$$

$$y = \begin{bmatrix} 1 & 3 \end{bmatrix} \mathbf{x}; \quad \mathbf{x}(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

EXPERIMENT [3,4]..EE362

OBJECTIVE

1- Evaluate the effect of pole and zero location upon the time response of first- and second-order systems.
2- Evaluate the effect of additional poles and zeros upon the time response of second-order systems.

MINIMUM REQUIRED SOFTWARE PACKAGES MATLAB, Simulink, and the Control System Toolbox.

PART 1

PRELAB

1- Given the transfer function $G(s) = \frac{a}{s+a}$ evaluate settling time and rise time for the following values of a: 1, 2, 3, 4. Also, plot the poles.

2- Given the transfer function $G(s) = \frac{b}{s^2+as+b}$.

a- Evaluate percent overshoot, settling time, peak time, and rise time for the following values: a = 4, b = 25. Also, plot the poles.

b- Calculate the values of a and b so that the imaginary part of the poles remains the same but the real part is increased two times over that of Prelab 2a, and repeat Prelab 2a.

c. Calculate the values of a and b so that the imaginary part of the poles remains the same but the real part is decreased (1/2) time over that of Prelab 2a, and repeat Prelab 2a.

3. a. For the system of Prelab 2a, calculate the values of a and b so that the real part of the poles remains the same but the imaginary part is increased two times over that of Prelab 2a, and repeat Prelab 2a.

b. For the system of Prelab 2a, calculate the values of a and b so that the real part of the poles remains the same but the imaginary part is increased four times over that of Prelab 2a, and repeat Prelab 2a.

4. a. For the system of Prelab 2a, calculate the values of a and b so that the damping

ratio remains the same but the natural frequency is increased two times over that of Prelab 2a, and repeat Prelab 2a.

b. For the system of Prelab 2a, calculate the values of a and b so that the damping ratio remains the same but the natural frequency is increased four times over that of Prelab 2a, and repeat Prelab 2a.

5. Briefly describe the effects on the time response as the poles are changed in each of Prelab 2, 3, and 4.

LAB

1. Using Simulink, set up the systems of Prelab 1 and plot the step response of each of the four transfer functions on a single graph by using the Simulink LTI Viewer. Also, record the values of settling time and rise time for each step response.

2. Using Simulink, set up the systems of Prelab 2. Using the Simulink LTI Viewer, plot the step response of each of the three transfer functions on a single graph. Also, record the values of percent overshoot, settling time, peak time, and rise time for each step response.

3. Using Simulink, set up the systems of Prelab 2a and Prelab 3. Using the Simulink LTI Viewer, plot the step response of each of the three transfer functions on a single graph. Also, record the values of percent overshoot, settling time, peak time, and rise time for each step response.

4. Using Simulink, set up the systems of Prelab 2a and Prelab 4. Using the Simulink LTI Viewer, plot the step response of each of the three transfer functions on a single graph. Also, record the values of percent overshoot, settling time, peak time, and rise time for each step response.

POSTLAB

1. For the first-order systems, make a table of calculated and experimental values of settling time, rise time, and pole location.

2. For the second-order systems of Prelab 2, make a table of calculated and experimental values of percent overshoot, settling time, peak time, rise time, and pole location.

3. For the second-order systems of Prelab 2a and Prelab 3, make a table of calculated and experimental values of percent overshoot, settling time, peak time, rise time, and pole location.

4. For the second-order systems of Prelab 2a and Prelab 4, make a table of calculated and experimental values of percent overshoot, settling time, peak time, rise time, and pole location.

5. Discuss the effects of pole location upon the time response for both first- and second-order systems. Discuss any discrepancies between your calculated and experimental values.

PART 2

PRELAB

1. a. Given the transfer function $G(s) = \frac{25}{s^2+4s+25}$ evaluate the percent overshoot, settling time, peak time, and rise time. Also, plot the poles.

b. Add a pole at -200 to the system of Prelab la. Estimate whether the transient response in Prelab la will be appreciably affected.

c. Repeat Prelab lb with the pole successively placed at -20, - 10, and -2.

2. A zero is added to the system of Prelab la at -200 and then moved to -50, - 20, - 10, - 5, and -2. List the values of zero location in the order of the greatest to the least effect upon the pure second-order transient response.

3- Given the transfer function $G(s) = \frac{\left(\frac{25b}{a}\right)(s+a)}{(s+b)(s^2+4s+25)}$, let a = 3 and b = 3.01, 3.1, 3.3, 3.5, and 4.0. Which values of b will have minimal effect upon the pure second-order transient response?

4- Given the transfer function $G(s) = \frac{\left(\frac{2500b}{a}\right)(s+a)}{(s+b)(s^2+40s+2500)}$, let a = 30 and b = 30.01, 30.1, 30.5, 31 , 35 and 40.0. Which values of b will have minimal effect upon the pure second-order transient response?

LAB

1. Using Simulink, add a pole to the second-order system of Prelab la and plot the step responses of the system when the higher-order pole is nonexistent, at -200, - 20, - 10, and -2. Make your plots on a single graph, using the Simulink

LTI Viewer. Normalize all plots to a steady-state value of unity. Record percent overshoot, settling time, peak time, and rise time for each response.

2. Using Simulink, add a zero to the second-order system of Prelab 1 a and plot the step responses of the system when the zero is nonexistent, at -200, - 50,-20, - 10, - 5, and -2. Make your plots on a single graph, using the Simulink LTI Viewer. Normalize all plots to a steady-state value of unity. Record percent overshoot, settling time, peak time, and rise time for each response.

3. Using Simulink and the transfer function of Prelab 3 with a = 3, plot the step responses of the system when the value of b is 3, 3.01, 3.1, 3.3, 3.5, and 4.0. Make your plots on a single graph using the Simulink LTI Viewer. Record percent overshoot, settling time, peak time, and rise time for each response.

4. Using Simulink and the transfer function of Prelab 4 with a = 30, plot the step responses of the system when the value of b is 30, 30.01, 30.1, 30.5, 31, 35, and 40. Make your plots on a single graph, using the Simulink LTI Viewer.Record percent overshoot, settling time, peak time, and rise time for each response.

POSTLAB

1. Discuss the effect upon the transient response of the proximity of a higher-order pole to the dominant second-order pole pair.

2. Discuss the effect upon the transient response of the proximity of a zero to the dominant second-order pole pair. Explore the relationship between the length of the vector from the zero to the dominant pole and the zero's effect upon the pure second-order step response.

3. Discuss the effect of pole-zero cancellation upon the transient response of a dominant second-order pole pair. Allude to how close the canceling pole and zero should be and the relationships of (1) the distance between them and (2) the distance between the zero and the dominant second-order poles.

EXPERIMENT [3,4]……………………………………………EE362

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4. a. For the system of Prelab 2a, calculate the values of a and b so that the damping

ratio remains the same but the natural frequency is increased two times over that of Prelab 2a, and repeat Prelab 2a.

b. For the system of Prelab 2a, calculate the values of a and b so that the damping ratio remains the same but the natural frequency is increased four times over that of Prelab 2a, and repeat Prelab 2a.

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2. Using Simulink, set up the systems of Prelab 2. Using the Simulink LTI Viewer, plot the step response of each of the three transfer functions on a single graph. Also, record the values of percent overshoot, settling time, peak time, and rise time for each step response.

3. Using Simulink, set up the systems of Prelab 2a and Prelab 3. Using the Simulink LTI Viewer, plot the step response of each of the three transfer functions on a single graph. Also, record the values of percent overshoot, settling time, peak time, and rise time for each step response.

4. Using Simulink, set up the systems of Prelab 2a and Prelab 4. Using the Simulink LTI Viewer, plot the step response of each of the three transfer functions on a single graph. Also, record the values of percent overshoot, settling time, peak time, and rise time for each step response.

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2. For the second-order systems of Prelab 2, make a table of calculated and experimental values of percent overshoot, settling time, peak time, rise time, and pole location.

3. For the second-order systems of Prelab 2a and Prelab 3, make a table of calculated and experimental values of percent overshoot, settling time, peak time, rise time, and pole location.

4. For the second-order systems of Prelab 2a and Prelab 4, make a table of calculated and experimental values of percent overshoot, settling time, peak time, rise time, and pole location.

5. Discuss the effects of pole location upon the time response for both first- and second-order systems. Discuss any discrepancies between your calculated and experimental values.

PART 2

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1. a. Given the transfer function $G(s) = \frac{25}{s^2+4s+25}$ evaluate the percent over-shoot, settling time, peak time, and rise time. Also, plot the poles.

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3- Given the transfer function $G(s) = \frac{\left(\frac{25b}{a}\right)(s+a)}{(s+b)(s^2+4s+25)}$, let a = 3 and b = 3.01, 3.1, 3.3, 3.5, and 4.0. Which values of b will have minimal effect upon the pure second-order transient response?

4- Given the transfer function $G(s) = \frac{\left(\frac{2500b}{a}\right)(s+a)}{(s+b)(s^2+40s+2500)}$, let a = 30 and b = 30.01, 30.1, 30.5, 31 , 35 and 40.0. Which values of b will have minimal effect upon the pure second-order transient response?

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3. Using Simulink and the transfer function of Prelab 3 with a = 3, plot the step responses of the system when the value of b is 3, 3.01, 3.1, 3.3, 3.5, and 4.0. Make your plots on a single graph using the Simulink LTI Viewer. Record percent overshoot, settling time, peak time, and rise time for each response.

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